

DAMTP-2011-2
January 14, 2011

A lattice calculation of $B \rightarrow K^{(*)}$ form factors¹

ZHAOFENG LIU^{a2}, STEFAN MEINEL^b, ALISTAIR HART^c, RON R. HORGAN^a,
EIKE H. MÜLLER^c, MATTHEW WINGATE^a

^a*DAMTP, University of Cambridge, Wilberforce Road, Cambridge CB3 0WA, UK*

^b*Department of Physics, College of William & Mary, Williamsburg, VA 23187-8795, USA*

^c*SUPA, School of Physics and Astronomy, University of Edinburgh, Edinburgh EH9 3JZ, UK*

Lattice QCD can contribute to the search for new physics in $b \rightarrow s$ decays by providing first-principle calculations of $B \rightarrow K^{(*)}$ form factors. Preliminary results are presented here which complement sum rule determinations by being done at large q^2 and which improve upon previous lattice calculations by working directly in the physical b sector on unquenched gauge field configurations.

PRESENTED AT

The 6th International Workshop on the CKM Unitarity
Triangle
University of Warwick, UK, 6-10 September 2010

¹Work supported by the UK Science and Technology Facilities Council.

²Speaker. Current address: Institute of High Energy Physics, Chinese Academy of Sciences, Beijing 100049, China.

1 Introduction

The $b \rightarrow s$ flavour changing neutral current transition is suppressed in the standard model. Dominant contributions to rare B decays such as $B \rightarrow K^{(*)}l^+l^-$ come from loop diagrams: box and penguin diagrams. These rare B decays are good windows for looking for new physics: New particles beyond the standard model could appear in the loops and change the decay widths of those rare decays.

The starting point of theoretical calculations of weak decays of hadrons is the effective weak Hamiltonian. In the standard model, there are ten operators in the effective Hamiltonian for radiative and semileptonic decays. The dominant short distance contributions are from effective local operators Q_7 , Q_9 and Q_{10} , which come from the penguin and box diagrams.

In quantum chromodynamics (QCD), quarks are confined in color singlets. The $b \rightarrow s$ transition happens inside hadrons. Therefore the matrix elements of the above three local operators have to be computed using non-perturbative methods, for example, lattice QCD. Those matrix elements can be parametrized by form factors according to their Lorentz structures. In total, there are ten form factors for the quark currents in Q_7 , Q_9 , and Q_{10} , and our aim is to calculate these on the lattice with dynamical simulations.

More details of our calculation strategy and our definitions of the form factors can be found in Ref. [1]. Here we update our progress in the extraction of the form factors. In the next section, we present our lattice setup and then we show some preliminary results in the last section.

2 Lattice setup

We use configurations from the MIMD* Lattice Computation (MILC) Collaboration, which are $2 + 1$ flavour dynamical simulations using $\mathcal{O}(a^2)$ and tadpole improved staggered fermions (AsqTad) [2]. We currently have data from two lattice spacings.

	$a(\text{fm})$	am_{sea}	Volume	$N_{conf} \times N_{src}$	am_{val}
coarse	~ 0.12	0.007/0.05	$20^3 \times 64$	2109×8	0.007/0.04
		0.02/0.05	$20^3 \times 64$	2052×8	0.02/0.04
fine	~ 0.09	0.0062/0.031	$28^3 \times 96$	1910×8	0.0062/0.031

Table 1: Parameters of lattices being used in this study. N_{src} is the number of point sources used on each configuration.

At the coarse lattice spacing we have two different light quark masses. The lightest

*Multiple Instruction stream, Multiple Data stream.

quark mass gives a pion mass of about 300 MeV. The parameters of our calculation are collected in Table 1. On each configuration, eight point sources are used to increase statistics. In [1], we found $Z_2 \times Z_2$ random wall sources were inefficient for vector mesons and heavy-light mesons in reducing statistical errors of correlators (random wall source methods allow one to approximately obtain all to all correlators and thus possibly to reduce statistical errors). Therefore we now use several point sources.

The light valance quarks are also AsqTad fermions as the sea quarks. For the heavy b quark, we use the (moving-)non-relativistic QCD (NRQCD) action [3], which is expanded up to and including $\mathcal{O}(\Lambda_{QCD}^2/m_b^2)$. We can work directly at the physical b quark mass, thus no extrapolation up to m_b is required. In the lattice heavy-light currents, the heavy quark expansion includes order $1/m_b$.

We compute 2-point functions for the heavy-light B meson and the light-light final state mesons as well as 3-point functions with the current operators inserted. Then we fit these correlation functions with the Bayesian fitting method [4]. From the fitted ground state energies and amplitudes, one can extract the matrix elements of the current operators and then the form factors. The detailed formulas can be found in Refs. [1, 5].

Matching factors of the lattice current operators to the $\overline{\text{MS}}$ scheme were computed perturbative in Ref. [6] to one loop. We set $\alpha_s = 0.3$ below to get the values of these matching factors.

Previous lattice calculations of the above form factors are all quenched calculations. And an extrapolation up to the physical b quark mass is needed for the heavy quark. See, e.g., [7] and the references therein.

3 Preliminary results

We tried both separate fitting and simultaneous fitting of the 2- and 3-point functions. The simultaneous fitting, where common energy parameters are used, gives better results with smaller statistical errors. Examples are shown in Fig. 1 for the ground state amplitude in a 3-point function for $B \rightarrow K$ and the B meson energy (in lattice units) obtained from different fits. Note a mass shifting term needs to be added for the B meson energy due to the use of (moving-)NRQCD. In Fig. 1 the horizontal axis is the number of (ground and excited) states in the Bayesian fitting functions. Equal number of opposite parity states are also in the fitting functions due to the use of staggered fermions. The fitting results stabilize when more than 6 normal states and 6 opposite parity states are used.

Preliminary results of some form factors for $B \rightarrow K$ and $B \rightarrow K^*$ are given in Fig. 2 and Fig. 3 respectively. They are from the coarse lattice with light valance quark masses 0.007/0.04. The $1/m_b$ corrections to the currents are not included yet, but will be soon. A first fitting to the 3-point functions shows that these corrections

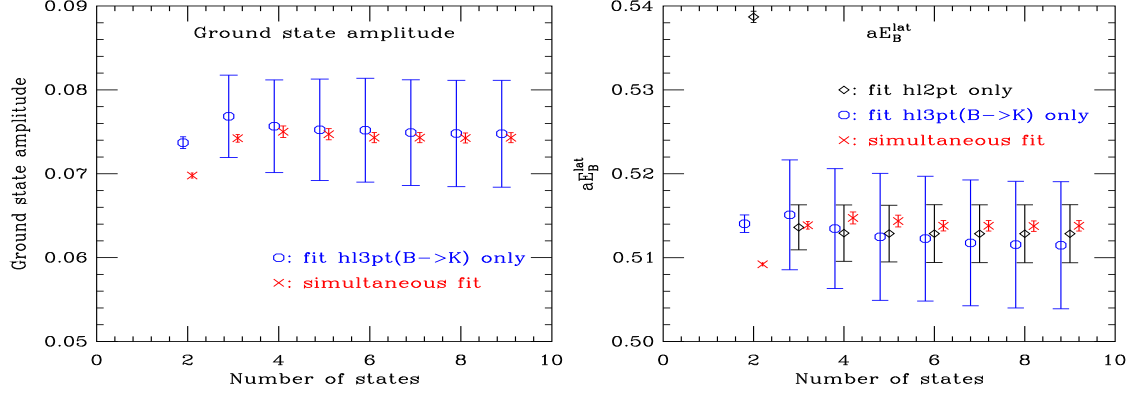


Figure 1: Comparison of separate Bayesian fitting and simultaneous Bayesian fitting to heavy-light 2-point (hl2pt) and heavy-light 3-point (hl3pt) functions for the ground state amplitude in a 3-point function (left graph) and the B meson energy aE_B^{lat} (right graph) in lattice units. The horizontal axis is the number of (ground and excited) states in the fitting functions.

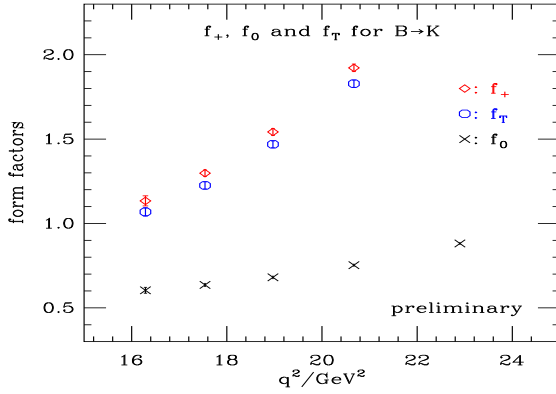


Figure 2: Preliminary results for the form factors f_0 , f_+ and f_T for $B \rightarrow K$ decays, obtained from simultaneous Bayesian fits. The left-most points have $\vec{v} = (0,0,0)$, $\vec{p}_B = 0$ and $\vec{p}_K = 2\pi/L \cdot (-2,0,0)$.

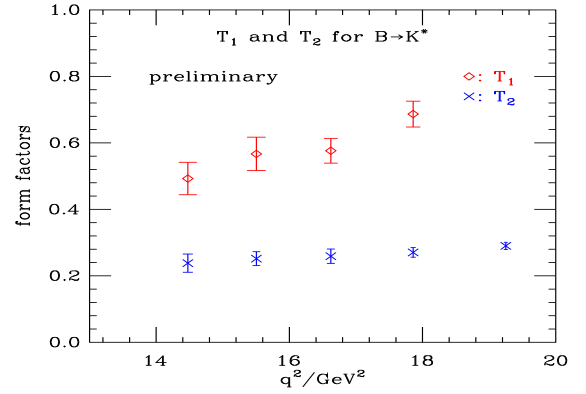


Figure 3: Preliminary results for the form factor T_1 and T_2 for $B \rightarrow K^*$ decays, obtained from simultaneous Bayesian fits. The left-most points have $\vec{v} = (0,0,0)$, $\vec{p}_B = 0$ and $\vec{p}_{K^*} = 2\pi/L \cdot (-2,0,0)$.

are small. For the other quark mass and for the fine lattice spacing, data analysis is going on. We will have more data points at lower q^2 after we analyze the correlators with non-zero velocity \vec{v} for the B meson in the moving NRQCD action.

For the radiative decay $B \rightarrow \gamma K^*$, we want the form factor $T(0) = T_1(0) = T_2(0)$ to get the branching fraction. To extrapolate our results to the $q^2 = 0$ limit, we need some ansatz. In Fig. 4 we do an extrapolation using the B-K ansatz [8]

$$T_1(q^2) = \frac{T(0)}{(1 - \tilde{q}^2)(1 - \alpha\tilde{q}^2)}, \quad T_2(q^2) = \frac{T(0)}{1 - \tilde{q}^2/\beta}, \quad \tilde{q}^2 = q^2/M_{B_s^*}^2, \quad (1)$$

with $M_{B_s^*} = 5.4158$ GeV fixed. This is only preliminary and we get $T(0) = 0.168(29)$.

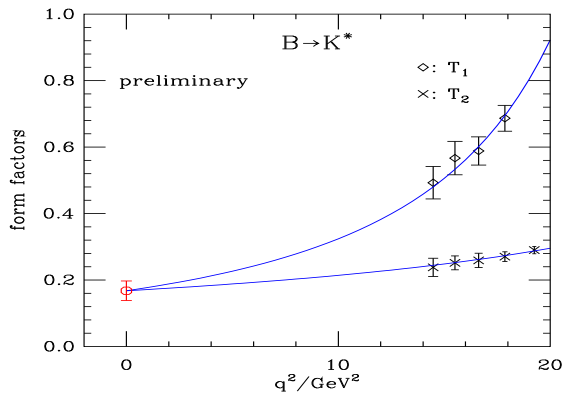


Figure 4: Extrapolation of T_1 and T_2 to $q^2 = 0$ using the B-K ansatz in Eq.(1).

The statistical errors will be reduced when more data points at low q^2 are included.

Eventually we need to extrapolate to the physical pion mass and the continuum limit to really compare with experiments. In a recent paper by H. Na et al.[9] on $D \rightarrow K$ form factors, the extrapolation in q^2 , light quark mass and lattice spacing were done all together using a “simultaneous modified z -expansion”. We may try this method in the future.

References

- [1] Z. Liu, S. Meinel, A. Hart, R. R. Horgan, E. H. Muller and M. Wingate, PoS **LAT2009** (2009) 242 [arXiv:0911.2370 [hep-lat]].
- [2] A. Bazavov *et al.*, Rev. Mod. Phys. **82** (2010) 1349 [arXiv:0903.3598 [hep-lat]].
- [3] R. R. Horgan *et al.*, Phys. Rev. D **80**, 074505 (2009) [arXiv:0906.0945 [hep-lat]].

- [4] G. P. Lepage, B. Clark, C. T. H. Davies, K. Hornbostel, P. B. Mackenzie, C. Morningstar and H. Trottier, Nucl. Phys. Proc. Suppl. **106**, 12 (2002) [[arXiv:hep-lat/0110175](#)].
- [5] S. Meinel, E. H. Muller, L. Khomskii, A. Hart, R. R. Horgan and M. Wingate, PoS **LATTICE2008** (2008) 280 [[arXiv:0810.0921 \[hep-lat\]](#)].
- [6] E. H. Müller, A. Hart and R. R. Horgan, [arXiv:1011.1215 \[hep-lat\]](#).
- [7] D. Becirevic, V. Lubicz and F. Mescia, Nucl. Phys. B **769** (2007) 31 [[arXiv:hep-ph/0611295](#)].
- [8] D. Becirevic and A. B. Kaidalov, Phys. Lett. B **478** (2000) 417 [[arXiv:hep-ph/9904490](#)].
- [9] H. Na, C. T. H. Davies, E. Follana, G. P. Lepage and J. Shigemitsu, [arXiv:1008.4562 \[hep-lat\]](#).